# A Fuzzy Model for an Automotive Engines-Development on a Production Vehicle

# نموذج مبنى على استخدام المنطق المشوش في محاكاة المحركات الخاصة بالمركبات

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# ملخص البحث

يقدم هذا البحث نموذجا مبنيا على استخدام الدنطق المشوش تم تنفيذه لمحرك سيارة. و يتم تمثيل هذا النموذج باستخدام قاعدة جمل شرطيه باسلوب تاكلجي- سوجينو. وقد تم تطبيق خوارزم جوستافسن لتحديد دوال العلاقة الخاصة بالجمل الشرطية المستخدمة. كما تم استخدام أسلوب أقل تقدير مربع لتحديد فعل الشرط في تلك الجمل، و قد تم اختيار أداء هذا النموذج باستخدام مجموعة قياسات تم تسجيلها من وحدة اختيار لها أربع سرعات تتغير أوتوماتيكيا. وقد تم جمع البياتات الخاصة بالمحاكاة من خلال مشروع تم تتفيذة مع جامعة جورج ميسون بامريكا. وبدر اسة النتائج المستخلصة فأنة قد تبين أن النموذج الذي تم تطويره و تنفيذه قادر على توفير المكانيات محاكاة جيدة.

#### Abstract

In this paper, we developed a fuzzy model for an automotive engine. The fuzzy model was presented by a set of rules based on the Takagi-Sugeno type. The Gustafson-Kessel (GK) algorithm was applied to determine the antecedent membership functions and east-square estimation was used to determine the consequence parameters. The performance of the fuzzy model was tested using a set of measurements recorded from a single production vehicle with a 3.1 I V-6 engine and a four speed automatic transmission. This data was collected earlier during a project implemented at George Mason University, USA. The developed fuzzy model was able to provide a good modeling capabilities.

#### 1. Introduction

The identification process for the dynamics of linear system is well defined. Mean while, the identification of nonlinear systems is a challenging task. The identification process of a complex nonlinear system can be considered as the development of a relationship between some input and output variables of the system under consideration. This is why the identification of a suitable model for industrial processes is a major problem for control engineering [1,2]. Finding a suitable model for an automobile engine depends mainly on the type of existing nonlinearity and the approach to which the model parameters are estimated [3,4]. Traditional approaches for structure determination and parameter identification have difficulty in estimating nonlinear system parameters especially with limited number of measurements [5].

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Fuzzy logic was originally introduced as a way to formally describing and manipularing linguistic information [6,7,8]. Later, it was clear that fuzzy logic is also a powerful tool for system identification and control of dynamical nonlinear processes [9,10,11]. In this paper we concentrate on the approximation of an automotive engine dynamics by a set of local linear models. Each local model is valid for a certain range of operating conditions and an interpolative scheduling mechanism combines the outputs of the local models into a continuous global output. Such a model structure can be conveniently represented by means of fuzzy If-Then rules. Using membership functions, the antecedent of the rule defines a fuzzy region in the product space of the antecedent variables in which the rule is valid. The antecedent variables must convey information about the process operating conditions. The consequent of the rule is typically a local linear regression model. The overlap of the antecedent membership functions of different rules provides a smooth interpolation of the rules' consequents.

#### 2. Engine System

The engine system has three inputs, throttle position is measured while fuel and exhaust gas recirculation are controlled. The block diagram of the engine system to be monitored is shown in Figure 1.

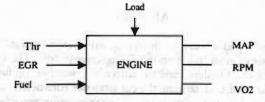


Figure 1: The engine variables

The developed modeling algorithm depends on the following variables:

- · fuel injectors (Fuel)
- exhaust gas recirculation valve (EGR)

and four sensors, namely

- throttle position (Thr)
- manifold pressure (MAP)
- engine speed (RPM)
- exhaust oxygen (VO2)

The engine system can be presented in the form of three single-output subsystems interacting to form the whole engine system.

- The Mainfold Subsystem contains the gas mechanics of the intake manifold, including the engine as a pump and the throttle, the ENG valve and the fuel injectors as input. Its output is the manifold absolute pressure.
- The Intertail Subsystem contains the dynamics of the movement of the
  powertrain and the vehicle. These dynamics depend on the vehicle mass, air drag,
  transmission gear, etc. The subsystem inputs are throttle, EGR, fuel, the
  manifold absolute pressure and the load torque; its output is the engine speed
  (RPM).
- 3. The Air-Fuel Subsystem contains the reaction chemistry of the engine. Its inputs are throttle, EGR, fuel, manifold absolute pressure and the engine speed, its output is the oxygen sensor voltage. Since only the internal subsystem is affected by the load torque (the unknown disturbance) and the time varying parameters such as the vehicle mass, insensitivity with respect to those can easily be achieved by omitting this subsystem from the modeling algorithm.

The function model of the engine can be presented in the form of three interacting single output subsystems as shown in Figure 2.

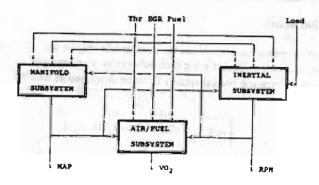


Figure 2: Engine subsystems

### 3. Fuzzy Model Structure

Many classes of nonlinear systems were modeled using the Takagi-Sugeno (TS) fuzzy models [12,13]. In our case, consider the dynamics of the engine can be described as input and output model. This will help in predicting the next model output. In the discrete-time system we can write the relationship between a system that has four inputs  $u_1(k).u_2(k).u_3(k).u_4(k)$  and single-output y(k) at time k in the following format:

$$y(k) = f(u_1(k), u_2(k), u_3(k), u_4(k))$$

The function f is a static function that relates the input and output of the engine model. Fuzzy models of different types can be used to approximate this relationship function f. One of the most common models is the NARX (Nonlinear Auto-Regressive with eXogenous input) model:

$$y(k) = f(u_1(k), u_2(k), ...., u_i(k))$$

(2)

 $u_1(k),...,u_i(k)$  and y(k) represents the model inputs and output, respectively. i is integer related to the model order.

For subsystem 1 of the engine, which has four inputs and single output, the set of fuzzy rules can be presented as follows:

$$R_i$$
: If  $u_i(k)$  is  $A_i$ , and .... and  $u_i(k)$  is  $A_i$ , then  $y(k)$  is  $c_i$ 

(3)

For subsystem 2, which has three inpurs and single output, the ser of fuzzy rules can be presented as follows:

$$R_i$$
: If  $u_i(k)$  is  $A_i$ , and ..., and  $u_i(k)$  is  $A_i$ , then  $y(k)$  is  $c_i$ 

(4)

Since fuzzy models can approximate any smooth function to any degree of accuracy [14] models of the type NARX can approximate any observable and controllable models of a large class of discrete-time nonlinear systems [15].

#### 4. Regression Matrix

Using the set of measurements N for the automobile engine we can build the regression matrix  $\phi$  and the output vector y for each subsystem. For example, the regression matrix  $\phi$  and the output vector y for subsystem 1 can be described as follows:

$$\phi = \begin{pmatrix} u_1(k) & u_2(k) & u_3(k) & u_4(k) \\ u_1(k+1) & u_2(k+1) & u_3(k+1) & u_4(k+1) \\ \vdots & \vdots & \ddots & \vdots \\ u_1(N) & u_2(N) & u_3(N) & u_4(N) \end{pmatrix}$$

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(5)

$$y = \begin{pmatrix} y(k) \\ y(k+1) \\ \vdots \\ \vdots \\ y(N) \end{pmatrix}$$

## 5. Identification Methodology

The structure of the model takes in consideration the user *a priori* knowledge about the system. Comparing several candidate structures in terms of the prediction error or other selected criteria [16] can be considered in our case.

Once the model structure selected, the next step is to estimate the main parameters of the fuzzy model. These parameters include the antecedent membership functions and the consequence polynomials. Additional parameter need to be selected. This parameter is the number of rules (clusters)  $\sigma$  which need to be specified by the user. The methodology to build a fuzzy model for modeling the dynamics of an automotive engine can be described in the following steps:

- 1. To develop the nonlinear regression model we collect a data set of measurements  $u_1(k), u_2(k), u_3(k), u_4(k)$  and the user defined parameters to find y(k).
- 2. Compute the antecedent membership function from the cluster parameters.
- Given the antecedent membership functions, estimate the consequence parameters by the least-square method.

This technique was introduced in [17,18] and was successfully applied to modeling and control of multi-input single output (MISO) system process [19,20] In the next section, we give some details about the identification methodology based fuzzy logic.

#### 5.1 Fuzzy Clustering

Given the regression matrix  $\phi$  and the specified number of clusters  $\sigma$ , Gustafson-Kessel (GK) algorithm [17,18] is applied. This algorithm computes the following:

- 1. The fuzzy partition matrix  $U = [\mu_{ik}]_{\sigma \times N}$  with  $\mu_{ik} \in [0,1]$ .  $\mu$  is the membership degree. i stand for the rule number.
- 2.  $V = [v_1, \dots, v_{\sigma}]$  is the prototype matrix.
- 3. The set of cluster covariance matrices  $F = [F_1, ..., F_{\sigma}]$ ,  $F_i$  are positive definite matrices in  $R^{(p+1)\times(p+1)}$ . p is the dimension of the antecedent space.

Given the triple, (U,V,F) the antecedent membership functions and the consequence parameters A, and c; can be computed for the two subsystems. The Gustafson-Kessel (GK) algorithm for multi-input multi-output (MIMO) systems, is described in [12,13,22].

## 5.2 Consequent Parameters

In our case, the fuzzy model inputs are  $\mathbf{u} = [u_n u_n u_n u_n]$ ,  $\mathbf{y}$  stand for the model output. There are several possibility to estimate the consequence parameters  $A_i$  and  $\epsilon_i$  as described in [22]. We adopted the weighted least-square estimation to find the fuzzy model parameters.

Let  $\theta^T$  be the vector which has the coefficient of the consequence polynomial  $A_i$  and  $c_k$ . Let  $\theta$  be the matrix [0,1] and the matrix W be a diagonal matrix with dimension  $R^{l\times l}$  having a membership degree  $\mu_k$  as its k th diagonal element. Assuming that the column of the matrix X are linearly independent and  $\mu_k > 0$  for  $1 \le k \le l$ , then:

$$\theta = (\phi^T W \phi)^{-1} \phi^T W y$$

(6)

 $\theta$  is the least-square solution of the equation  $y = X\theta + \delta$  where the k  $\underline{h}$  data pair (u,y) is weighted by  $\mu_k$ .

# 6. Experimental Setup

Experiments have been conducted on a single production vehicle with a 3.1 I V-6 engine and a four speed automatic transmission. The engine is equipped with simultaneous multi-port fuel injection and a three-stage binary EGR valve. The oxygen sensor in unheated. In addition to the basic actuators (Fuel, EGR) and sensors (Thr, MAP, RPM, VO2), the cat has a stepping motor driven idle air valve (IAC) and a coolant temperature and manifold air temperature sensor (COT, MAT). Data was collected, over several occasions, in the following operating modes:

- 1. city driving, normal car
- 2. highway driving, normal car
- 3. hilly terrain, normal car
- 4. hilly terrain, EGR valves stuck open

The training data set used is shown in Figure 3. A data set of 1000 measurements was used in the training case. A set of 2000 measurements that includes the training data was used in the testing process.

#### 6.1 Evaluation Criterion

As a figure of merit, we take the Variance-Accounted-For (VAF) as a major of performance in the modeling process. The VAF is calculated as:

$$VAF = 1 - \frac{\text{var}(y - y)}{\text{var}(y)} \times 100\%$$

(7)

#### 7. Modeling for Subsystem 1 of the Engine

#### 7.1 Regression Model

We developed a regression model for the sake of comparison. The model parameters were estimated using Least-Square Estimation (LSE). The equation that describes the dynamics of subsystem 1 was as follows:

$$y(k) = 0.2853u_1(k) - 0.2815u_2(k) + 0.0804u_3(k) + 0.0045u_4(k) + 1.6613$$

The VAF was computed in both training and testing cases using the above model. The actual output of subsystem 1 and the predicted output generated from the above model are shown in Figure 4 and 5, in both training and testing cases. The difference between the actual and estimated responses is also shown.

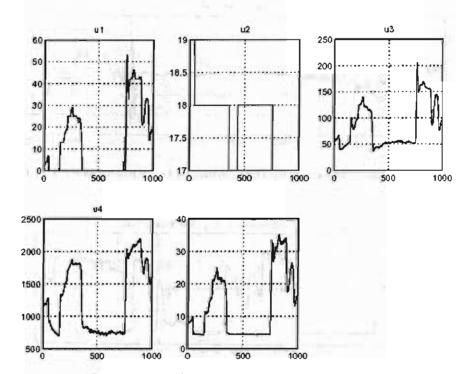


Figure 3: Training data set of  $u_1, u_2, u_3, u_4, y$  for subsystem no. 1

# 7.2 Fuzzy Model

To develop the fuzzy model for the engine we used the Fuzzy Model Identification Toolbox (FMID) written in Matlab [13]. To use the toolbox the input-output training data was described in a matrix format as in  $\varphi$  and y, respectively. The number of clusters  $\sigma$  should be identified by the user. In our case, we have tried number of clusters to build various models. The models with the best modeling capabilities will be selected.

In the case of subsystem1, the number of clusters was set to 4. This number is a scalar value, since we have a single output system. If we have more than one output, it should be set as a vector, which has an element for each output.

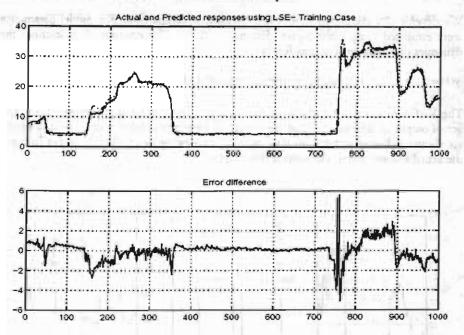


Figure 4: Actual and predicted responses for subsystem no. 1 in the training case: LSE case

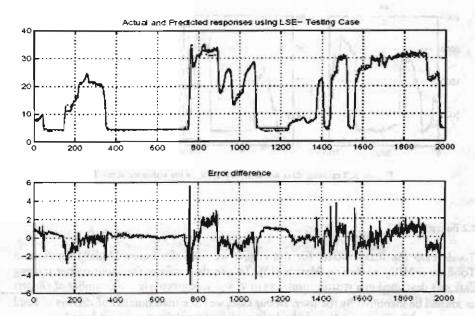


Figure 5: Actual and predicted responses for subsystem no. 1 in the testing case: LSE case

The remination tolerance for the clustering algorithm can be set priori. We used the developed fuzzy model to obtain the consequent parameters and the cluster centers. These parameters are shown in Table 1 and 2. In the following we show the set of rules that describe the developed fuzzy model for subsystem 1.

#### Rules:

- 1. If  $u_1$  is  $A_{11}$  and  $u_2$  is  $A_{12}$  and  $u_3$  is  $A_{13}$  and  $u_4$  is  $A_{14}$  then  $y(k) = 0.00 \times 10^0 u_1(k) 3.12 \times 10^{-1} u_2(k) + 5.06 \times 10^{-2} u_1(k) + 2.47 \times 10^{-3} u_4(k) + 5.55 \times 10^0$
- 2. If  $u_1$  is  $A_{21}$  and  $u_2$  is  $A_{22}$  and  $u_3$  is  $A_{23}$  and  $u_4$  is  $A_{24}$  then  $y(k) = 9.30 \times 10^{-2} u_1(k) 5.14 \times 10^{-2} u_2(k) + 1.40 \times 10^{-1} u_3(k) + 5.69 \times 10^{-4} u_4(k) + 0.00 \times 10^{0}$
- 3. If  $u_1$  is  $A_{31}$  and  $u_2$  is  $A_{32}$  and  $u_3$  is  $A_{33}$  and  $u_4$  is  $A_{34}$  then  $y(k) = 2.39 \times 10^{-1} u_1(k) 3.63 \times 10^{-1} u_2(k) + 1.01 \times 10^{-1} u_3(k) + 5.33 \times 10^{-3} u_4(k) + 0.00 \times 10^{0}$
- 4. If  $u_1$  is  $A_{41}$  and  $u_2$  is  $A_{42}$  and  $u_3$  is  $A_{43}$  and  $u_4$  is  $A_{44}$  then  $y(k) = 4.26 \times 10^{-1} \ u_1(k) 4.80 \times 10^{-1} u_2(k) + 4.25 \times 10^{-2} u_3(k) + 7.42 \times 10^{-3} u_4(k) + 0.00 \times 10^{0}$

rule	u <sub>I</sub>	u <sub>2</sub>	и <sub>3</sub>	u <sub>4</sub>	offset
1	$0.00 \times 10^{0}$	-3.12x10 <sup>-1</sup>	5.06x10 <sup>-2</sup>	2.47x10 <sup>-3</sup>	5.55x10°
2	9.30x10 <sup>-2</sup>	-5.14x10 <sup>-2</sup>	1.40x10 <sup>-1</sup>	5.69x10 <sup>-4</sup>	$0.00 \times 10^{0}$
3	2.39x10 <sup>-1</sup>	-3.63x10 <sup>-1</sup>	1.01x10 <sup>-1</sup>	5.33x10 <sup>-3</sup>	$0.00 \times 10^{\circ}$
4	4.26x10 <sup>-1</sup>	-4.80x10 <sup>-1</sup>	4.25x10 <sup>-2</sup>	7.42x10 <sup>-3</sup>	$0.00 \times 10^{\circ}$

Table 1: Consequence Parameters

rule	u <sub>l</sub>	u <sub>2</sub>	и3	14
1	2.14x10 <sup>-21</sup>	$1.78 \times 10^{1}$	5.04x10 <sup>1</sup>	$7.85 \times 10^{2}$
2	4.50x10°	1.90x10 <sup>1</sup>	6.05x10 <sup>1</sup>	$1.25 \times 10^3$
3	1.97x101	1.80x10 <sup>1</sup>	1.04x10 <sup>1</sup>	$1.54 \times 10^{3}$
4	3.44x10 <sup>1</sup>	1.70x10 <sup>1</sup>	$1.39 \times 10^{2}$	$1.89 \times 10^{3}$

Table 2: Clusters Centers

The actual output of subsystem 1 and the predicted output generated using the fuzzy model is presented in Figure 6 and 7, in both training and testing cases. The actual output is shown in the solid line and the generated output is shown in the dotted line. The error difference between the two characteristics is shown in the lower figures.

It can be seen from the results of modeling the dynamics of subsystem 1 using both regression and fuzzy models that the modeling abilities of the fuzzy model is better. We computed the VAF values in both the training and testing cases which are given in Table 5.

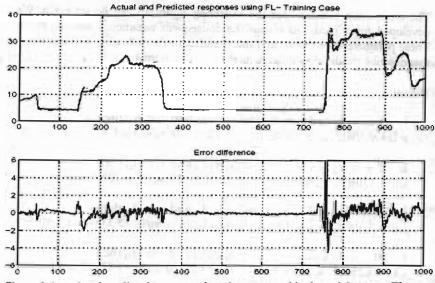


Figure 6: Actual and predicted responses for subsystem no. 1 in the training case: FL case

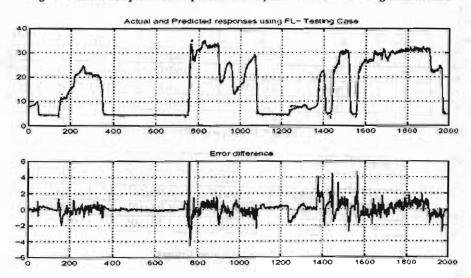


Figure 7: Actual and predicted responses for subsystem no. 1 in the testing case: FL case

# 8. Modeling for Subsystem 2 of the Engine

# 8.1 Regression Model

We developed a regression model for the sake of comparison. The model parameters were estimated using Least-Square Estimation (LSE). The equation that describes the regression model was found as follows:

$$y(k) = 5.7582u_1(k) - 0.1101u_2(k) - 0.0161u_3(k) + 37.9511$$

The set of model parameters were estimated using LSE. The training data used in our case is shown in Figure 8. The system has three inputs and single output. The actual and predicted responses in the regression case are shown in Figure 9 and 10.

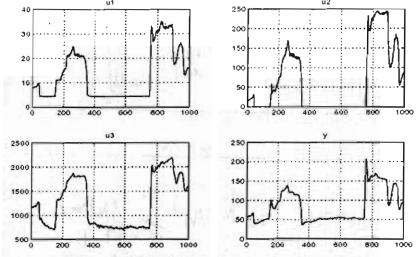


Figure 8: Training data set of  $u_1, u_2, u_3, y$  for subsystem no. 2

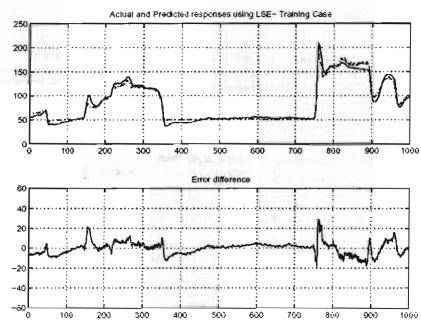
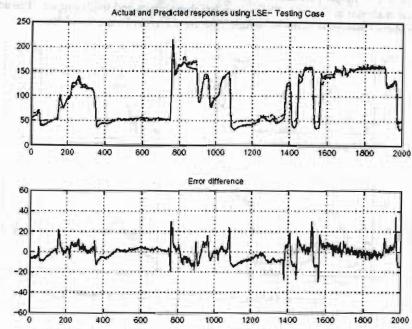


Figure 9: Actual and predicted responses for subsystem no. 2 in the training case: LSE case



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Figure 10: Actual and predicted responses for subsystem no. 2 in the testing case: LSE case

#### 8.2 Fuzzy Model

In this section we use the fuzzy modeling methodology to describe the dynamics of subsystem 2. The number of clusters  $\sigma$  selected to model in the case of subsystem 2 was set to 5. We used the developed fuzzy model to obtain the consequent parameters and the cluster centers. These parameters are shown in Table 3 and 4. In the following we show the set of rules that describe the developed fuzzy model for subsystem 2.

# Rules:

- 1. If  $u_1$  is  $A_{11}$  and  $u_2$  is  $A_{12}$  and  $u_3$  is  $A_{13}$  then  $y(k) = 5.35 \times 10^9 \ u_1(k) + 2.97 \times 10^9 u_2(k) 3.50 \times 10^{-2} u_3(k) + 5.4 \times 10^1$
- 2. If  $u_1$  is  $A_{21}$  and  $u_2$  is  $A_{22}$  and  $u_3$  is  $A_{23}$  then  $y(k) = 6.46 \times 10^0 \ u_1(k) + 2.06 \times 10^{-1} u_2(k) 8.22 \times 10^{-2} u_3(k) + 1.02 \times 10^2$
- 3. If  $u_1$  is  $A_{31}$  and  $u_2$  is  $A_{32}$  and  $u_3$  is  $A_{33}$  then  $y(k) = 4.44 \times 10^0 \ u_1(k) + 4.30 \times 10^{-1} u_2(k) + 2.05 \times 10^{-3} u_3(k) + 22.1 \times 10^0$
- 4. If  $u_1$  is  $A_{41}$  and  $u_2$  is  $A_{42}$  and  $u_3$  is  $A_{43}$  then  $y(k) = -1.03 \times 10^9 \ u_1(k) + 6.05 \times 10^{-1} u_2(k) 7.48 \times 10^{-3} u_3(k) + 75.6 \times 10^9$
- 5. If  $u_1$  is  $A_{51}$  and  $u_2$  is  $A_{52}$  and  $u_4$  is  $A_{53}$  then  $y(k) = 3.04 \times 10^9 \ u_1(k) + 4.15 \times 10^{-2} u_2(k) 7.33 \times 10^{-2} u_3(k) + 2.02 \times 10^2$

rule	u <sub>l</sub>	<i>u</i> <sub>2</sub>	113	offset
1	5.35x10°	2.97x10°	$-3.50 \times 10^{-2}$	5.4x10 <sup>1</sup>
2	6.46x10°	2.06x10 <sup>-1</sup>	-8.22x10 <sup>-2</sup>	$1.02 \times 10^2$
3	4.44x10°	4.30x10 <sup>-1</sup>	2.05x10 <sup>-3</sup>	$22.1 \times 10^{\circ}$
4	-1.03x10°	6.05x10 <sup>-1</sup>	$-7.48 \times 10^{-3}$	$75.6 \times 10^{0}$
5	$3.04 \times 10^{0}$	4.15x10 <sup>-2</sup>	$-7.33 \times 10^{-2}$	$2.02 \times 10^{2}$

Table 3: Consequence Parameters

rule	u <sub>l</sub>	u <sub>2</sub>	$u_3$
1	4.59x10°	1.30x10 <sup>-2</sup>	$7.79 \times 10^{2}$
2	1.26x101	5.31x10 <sup>1</sup>	$1.40 \times 10^{3}$
3	1.35x10 <sup>1</sup>	4.60x10 <sup>1</sup>	9.01x10 <sup>2</sup>
4	2.17x10 <sup>1</sup>	$1.36 \times 10^{2}$	$1.81 \times 10^{3}$
5	3.24x10 <sup>1</sup>	$2.34 \times 10^{2}$	2.07x103

Table 4: Clusters Centers

Figures 11 and 12 we show the actual and predicted responses for the fuzzy model of subsystem 2. The same set of training and testing data used in the regression case was used to test our fuzzy model.

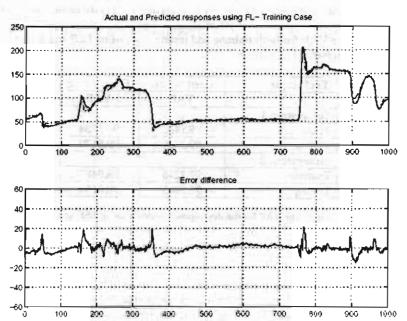


Figure 11: Actual and predicted responses for subsystem no. 2 in the training case: FL case

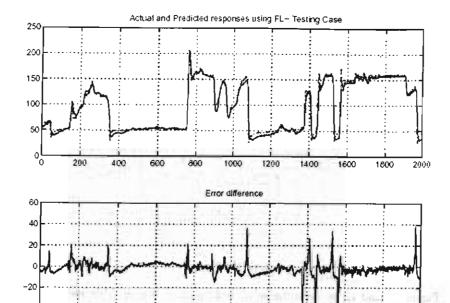


Figure 12: Actual and predicted responses for subsystem no. 2 in the testing case: FL case

The values of the VAF for both training and testing cases using LSE and Fuzzy logic techniques are presented in Table 5.

Technique	Least-Square	Fuzzy Logic
	VAF	VAF
Subsystem 1		- 415
Training	99.1462	99.5788
Testing	99.2469	99.5179
Subsystem 2		
Training	97.7248	98.9487
Testing	97.5905	98.0164

Table 5: The VAF for the development models using LSE and FL

# Conclusions

In this paper we developed a fuzzy model for two main subsystems of the automotive engine. The fuzzy model was presented by a set of rules based on the Takagi-Sugeno type. The Gustafson-Kessel (GK) algorithm was applied to determine the antecedent membership functions and least-square estimation was used to determine the consequence parameters. The performance of the fuzzy model was tested using a set of measurements recorded from a single production vehicle with a 3.1 I V-6 engine and a four speed automatic transmission. The performance of the fuzzy model was tested using the VAF criterion. The fuzzy model was successfully able to build a relationship between the model input and output. The results for both training and testing cases were better than the regression model in all cases.

#### Acknowledgments

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